落書き帳「616」について

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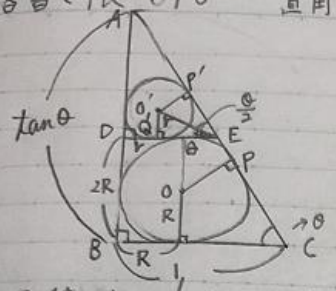
145 落書き帳 616 直角三角形の中の二円（その2）



頭円の半径 r，胴円の半径 R とする

$\triangle ABC$ の $\angle B = 90^\circ$　$\angle C = \theta$

$BC = 1$ とする

$AB = \tan\theta$

$AC = \dfrac{1}{\cos\theta}$　$\left(\cos\theta = \dfrac{BC}{AC}\right)$

・$\triangle ABC$ の面積より　$\dfrac{1}{2}\tan\theta = \left(1 + \tan\theta + \dfrac{1}{\cos\theta}\right)R \times \dfrac{1}{2}$ より

$$R(\sin\theta + \cos\theta + 1) = \sin\theta \quad ①$$

・$\triangle ABC \backsim \triangle ADE$ より　$AB : AD = BC : DE$

$$\tan\theta : \tan\theta - 2R = 1 : DE$$

$$\tan\theta \times DE = \tan\theta - 2R$$

$$DE = 1 - \frac{2R}{\tan\theta} \quad ②$$

・$\triangle ADE$ において　$DE = DQ' + EQ'$

$$= r + EQ'$$

また　$\tan\dfrac{\theta}{2} = \dfrac{r}{EQ'}$　　$\boxed{\tan^2\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{1+\cos\theta}}$

$$EQ' = \frac{r}{\tan\frac{\theta}{2}}$$

$$= \frac{r\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} = \frac{r(1+\cos\theta)}{\sin\theta}$$

$$DE = r\left(1 + \frac{1+\cos\theta}{\sin\theta}\right) = r \times \frac{\sin\theta + \cos\theta + 1}{\sin\theta} \quad ③$$

②＝③より　$1 - \dfrac{2R}{\tan\theta} = r \times \dfrac{\sin\theta + \cos\theta + 1}{\sin\theta}$

$$1 - \frac{2R\cos\theta}{\sin\theta} = \frac{r \times (\sin\theta + \cos\theta + 1)}{\sin\theta}$$

①を代入　$\sin\theta - 2R\cos\theta = r(\sin\theta + \cos\theta + 1)$

$$R(\sin\theta + \cos\theta + 1) - 2R\cos\theta = r(\sin\theta + \cos\theta + 1)$$

$$R(\sin\theta - \cos\theta + 1) = r(\sin\theta + \cos\theta + 1)$$

よって

$$r : R = \sin\theta - \cos\theta + 1 : \sin\theta + \cos\theta + 1$$